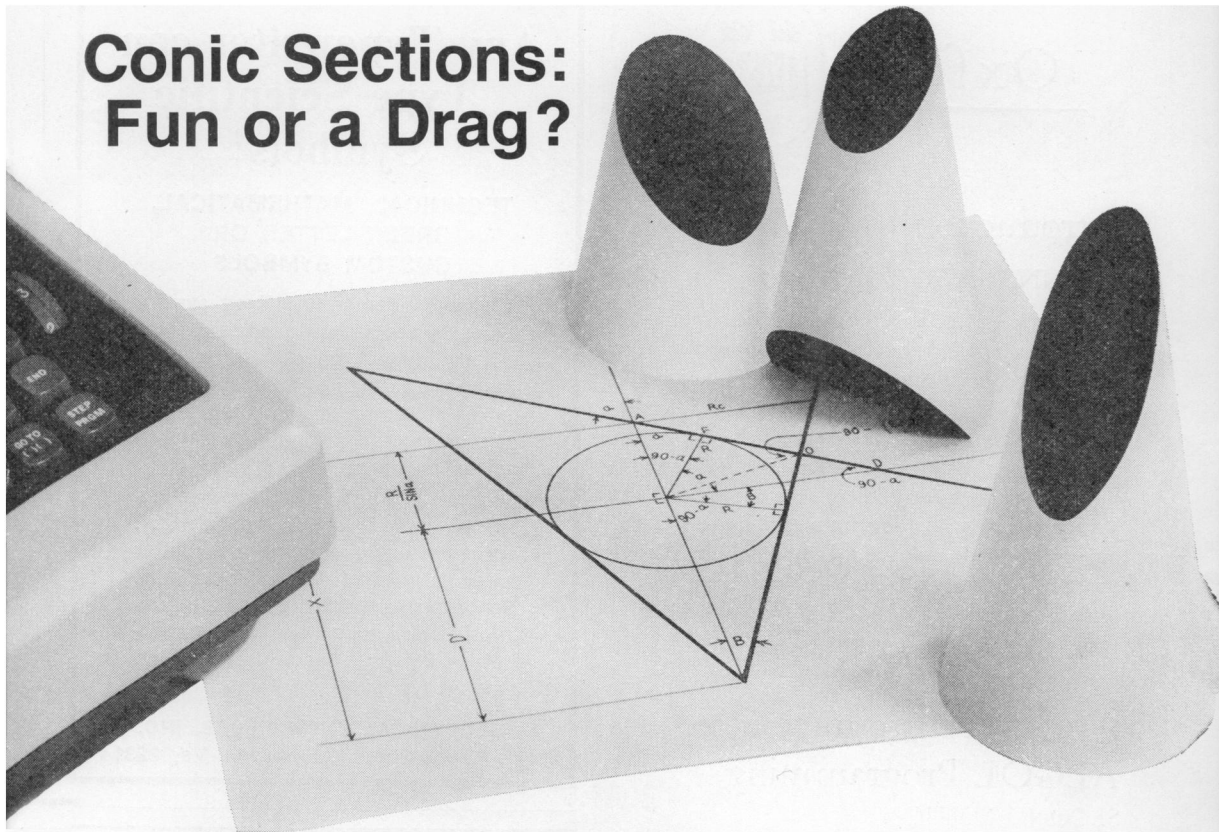


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Index of Papers and Technical Notes by Authors	899
Index of Reviews by Author of Work Reviewed	902
Index of Reviews by Subject of Work Reviewed	907
Index of Table Errata	918
Index of Corrigenda	919
Index of Microfiche Supplements	919

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Mathematics of Computation

TABLE OF CONTENTS

OCTOBER 1969

Convergence Estimates for Essentially Positive Type Discrete Dirichlet Problems . . . J. H. BRAMBLE, B. E. HUBBARD & VIDAR THOMÉE	695
Asymptotic Behavior of Solutions to the Finite-Difference Wave Equation CARL E. PEARSON	711
Finite-Difference Methods and the Eigenvalue Problem for Nonselfadjoint Sturm-Liouville Operators ALFRED CARASSO	717
Block Implicit One-Step Methods . . . L. F. SHAMPINE & H. A. WATTS	731
A Note on the Stability of Predictor-Corrector Techniques . . JAMES CASE	741
Stochastic Quadrature Formulas SEYMOUR HABER	751
Perfectly Symmetric Two-Dimensional Integration Formulas with Minimal Numbers of Points PHILIP RABINOWITZ & NIRA RICHTER	765
Eberlein Measure and Mechanical Quadrature Formulae. II. Numerical Results V. L. N. SARMA & A. H. STROUD	781
Stability Configurations of Electrons on a Sphere . . MICHAEL GOLDBERG	785
Extensions and Applications of the Householder Algorithm for Solving Linear Least Squares Problems RICHARD J. HANSON & CHARLES L. LAWSON	787
A Steepest Ascent Method for the Chebyshev Problem MARCEL MEICLER	813
Reducing a Matrix to Hessenberg Form P. A. BUSINGER	819
A Generalization of a Class of Test Matrices ROBERT J. HERBOLD	823
Nonnegative Matrix Equations Having Positive Solutions JERRY A. WALTERS	827
On Lehmer's Method for Finding the Zeros of a Polynomial G. W. STEWART III	829
The Solution of Integral Equations in Chebyshev Series . R. E. SCRATON	837
Integral Relations Among Bessel Functions . . . E. O. SCHULZ-DUBOIS	845
Some Limiting Cases of the G -Transformation H. L. GRAY & W. R. SCHUCANY	849
Factorization of Polynomials over Finite Fields . . ROBERT J. McELIECE	861
Lucasian Criteria for the Primality of $N = h \cdot 2^n - 1$. . . HANS RIESEL	869
Some New Results on Equal Sums of Like Powers . . . SIMCHA BRUDNO	877
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS	881
BERGER, DANSON & CARPENTER 60, COLLATZ, MEINARDUS & UNGER 59, COSRIMS 70, 71, 72, FETTIS & CASLIN 63, HUBBELL & CHRISTOFFERSEN 61, KELLY 58, LANCASTER 66, MIKSA 69, MURTY & TAYLOR 68, PATTERSON 62, RIORDAN 64, ROMAN 67, SPIEGEL 65, YODEN 57	
TABLE ERRATA	891
ABRAMOWITZ & STEGUN 444, ERDÉLYI, MAGNUS, OBERHETTINGER & TRICOMI 445, GRADSHTEYN & RYZHIK 446, LANZOS 447, PATTERSON 448, SPIEGEL 449	
CORRIGENDA	893
GUTSCHICK & LUDWIG, YANG	

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Gaussian Formulae for the Calculation of
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75240

It is well known that the repeated integral,

$$I = \int_{-1}^1 dx_1 \int_a^{x_1} dx_2 \dots \int_a^{x_{n-1}} f(x_n) dx_n$$

can be transformed to,

$$I = \frac{1}{(n-1)!} \int_{-1}^1 w(x) f(x) dx$$

where the weight function $w(x)$ is given by,

$$\begin{aligned} w(x) &= (1-x)^{n-1} & a \leq x \leq 1 \\ &= (-1-x)^{n-1} & -1 \leq x \leq a \end{aligned} \tag{2}$$

The highest precision formulae for the evaluation of the repeated integral are thus the Gaussian formulae appropriate to this weight function. The abscissae of the r point quadrature formula are the zeros of the polynomial $P_r(x)$ which is orthogonal with respect to $w(x)$ to all polynomials of degree less than r , that is

$$\int_{-1}^1 w(x) P_r(x) x^k dx = 0 \quad k=0, 1, \dots, r-1$$

Formulae for $a=0$ containing 2 up to 16 points have been computed for 2, 3, 4, and 5 dimensional repeated integrals, using a