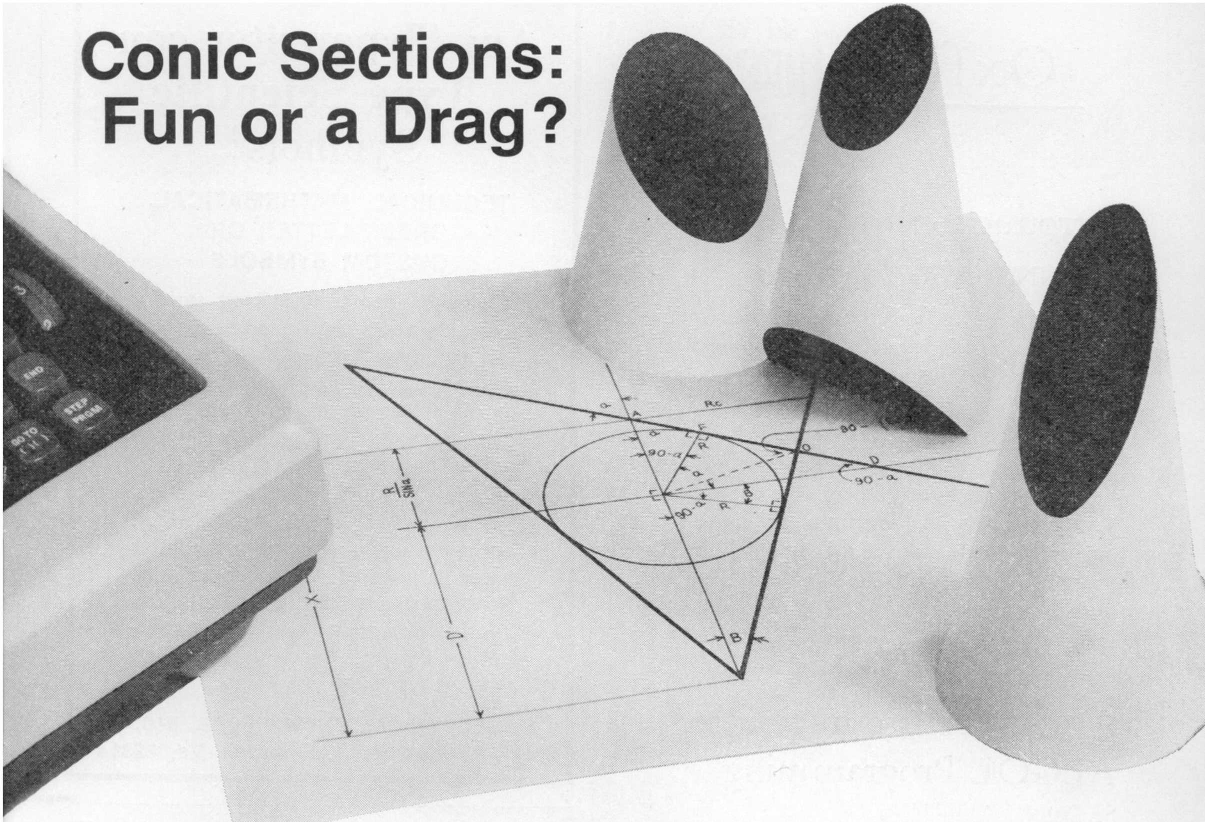
Conic Sections: Fun or a Drag?



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099/28

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Subject Classification System for Index of Reviews 895
Indices to Volume XXIII 899
Index of Papers and Technical Notes by Authors 899
Index of Reviews by Author of Work Reviewed 902
Index of Reviews by Subject of Work Reviewed 907
Index of Table Errata 918
Index of Corrigenda 919
Index of Microfiche Supplements 919

The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

Mathematics of Computation

TABLE OF CONTENTS

OCTOBER 1969

Convergence Estimates for Essentially Positive Type Discrete Dirichlet Problems . . . J. H. BRAMBLE, B. E. HUBBARD & VIDAR THOMÉE 695

Asymptotic Behavior of Solutions to the Finite-Difference Wave Equation CARL E. PEARSON 711

Finite-Difference Methods and the Eigenvalue Problem for Nonselfadjoint Sturm-Liouville Operators . . . . ALFRED CARASSO 717

Block Implicit One-Step Methods . . . L. F. SHAMPINE & H. A. WATTS 731

A Note on the Stability of Predictor-Corrector Techniques . . JAMES CASE 741

Stochastic Quadrature Formulas . . . . SEYMOUR HABER 751

Perfectly Symmetric Two-Dimensional Integration Formulas with Minimal Numbers of Points . . . . . PHILIP RABINOWITZ & NIRA RICHTER 765

Eberlein Measure and Mechanical Quadrature Formulae. II. Numerical Results . . . . V. L. N. SARMA & A. H. STROUD 781

Stability Configurations of Electrons on a Sphere . . MICHAEL GOLDBERG 785

Extensions and Applications of the Householder Algorithm for Solving Linear Least Squares Problems RICHARD J. HANSON & CHARLES L. LAWSON 787

A Steepest Ascent Method for the Chebyshev Problem MARCEL MEICLER 813

Reducing a Matrix to Hessenberg Form . . . . P. A. BUSINGER 819

A Generalization of a Class of Test Matrices . . . . ROBERT J. HERBOLD 823

Nonnegative Matrix Equations Having Positive Solutions JERRY A. WALTERS 827

On Lehmer's Method for Finding the Zeros of a Polynomial G. W. STEWART III 829

The Solution of Integral Equations in Chebyshev Series . R. E. SCRATON 837

Integral Relations Among Bessel Functions . . . E. O. SCHULZ-DUBOIS 845

Some Limiting Cases of the G-Transformation H. L. GRAY & W. R. SCHUCANY 849

Factorization of Polynomials over Finite Fields . . ROBERT J. MCELIECE 861

Lucasian Criteria for the Primality of $N = h \cdot 2^n - 1$ . . . HANS RIESEL 869

Some New Results on Equal Sums of Like Powers . . . SIMCHA BRUDNO 877

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS 881

BERGER, DANSON & CARPENTER **60**, COLLATZ, MEINARDUS & UNGER **59**, COSRIMS **70**, **71**, **72**, FETTIS & CASLIN **63**, HUBBELL & CHRISTOFFERSEN **61**, KELLY **58**, LANCASTER **66**, MIKSA **69**, MURTY & TAYLOR **68**, PATTERSON **62**, RIORDAN **64**, ROMAN **67**, SPIEGEL **65**, YOUDEN **57**

TABLE ERRATA 891

ABRAMOWITZ & STEGUN **444**, ERDÉLYI, MAGNUS, OBERHETTINGER & TRICOMI **445**, GRADSHTEYN & RYZHIK **446**, LANCZOS **447**, PATTERSON **448**, SPIEGEL **449**

CORRIGENDA 893

GUTSCHICK & LUDWIG, YANG

GAUSSIAN FORMULAE

FOR THE CALCULATION OF REPEATED INTEGRALS

BY

T.N.L. PATTERSON

Gaussian Formulae for the Calculation of Repeated Integrals, by T N L Patterson.

Southwest Center for Advanced Studies.
Dallas, Texas
75240

It is well known that the repeated integral,

$$I = \int_{-1}^{1} dx_1 \int_{a}^{x_1} dx_2 \qquad \int_{a}^{x_{n-1}} f(x_n)dx_n$$

can be transformed to,

$$I = \frac{1}{(n-1)!} \int_{-1}^{1} w(x)\ f(x)dx$$

where the weight function w(x) is given by,

$$\begin{aligned} w(x) &= (1-x)^{n-1} \qquad && a \leq x \leq 1 \\ &= (-1-x)^{n-1} && -1 \leq x \leq a \end{aligned} \qquad (2)$$

The highest precision formulae for the evaluation of the repeated integral are thus the Gaussian formulae appropriate to this weight function. The abscissae of the r point quadrature formula are the zeros of the polynomial $P_r(x)$ which is orthogonal with respect to w(x) to all polynomials of degree less than r, that is

$$\int_{-1}^{1} w(x)\ P_r(x)\ x^k\ dx=0 \quad k=0,1, \quad \ldots, \ r-1$$

Formulae for a=0 containing 2 up to 16 points have been computed for 2, 3, 4, and 5 dimensional repeated integrals, using a